To the Editor:

In "Validation of a New Filtration Technique for Dewaterability Characterization," (July 2001), Usher et al. reported cake filtration data with incrementally increasing applied pressure and presented a model for data interpretation. This model was also applied in a subsequent study (de Kretser et. al., 2001).

The model was similar to the conventional cake filtration theory with some modifications. The purpose of this letter is to present our concerns about the work and to offer an alternative approach.

Our reservations can be summarized as follows:

(1) The omission of the medium (membrane) resistance is questionable. Regardless of its magnitude, it is the dominant resistance initially since there is no cake at $t = 0$.

(2) The cake solidosity (ϕ) was assumed to be uniform and at a value corresponding to a compressive stress equal to the operating pressure. This is contrary to the characteristic feature of compressible cake filtration, namely, the nonuniformity of ϕ across the thickness of a cake.

(3) It was assumed that during cake compaction no suspended particles were added to the cake/suspension interface. This assumption was at variance with the manner with which the experiments were conducted.

This new model gives a linear relationship between t (time) and V^2 (V being the cumulative filtrate volume), the gradient of which can be used to estimate the specific cake resistance (or, in the authors' terminology, the hindered settling function R). By using this procedure, consistent values of R were obtained, which was construed as *a posteriori* validation of the model.

The writers are of the opinion that instead of developing a new model, the conventional cake theory could very well be used for the authors' purpose as a basis of their data treatment. For the clarity of subsequent discussions, Eq. 2 of Usher et al. may be rewritten as

$$\frac{dV}{dt} = \frac{P}{\eta[\overline{(w/V)(\alpha_{av})_{\Delta p_c}}V + R_m]} \quad (1)$$

where R_m is the medium resistance and w is the dry cake mass. α_{av} is the average specific cake resistance defined as

$$(\alpha_{av})_{\Delta p_c} = \frac{\Delta p_c}{\int_0^{p_{sm}} (h\epsilon\phi\rho_s)dp_s} \quad (2)$$

Δp_c is the pressure drop across the cake and p_{s_m} is the cake compressive stress p_s at the cake/medium interface. If one assumes $dp_\ell + dp_s = 0$ (p_ℓ pore liquid pressure), $p_{s_m} = \Delta p_c$.

For constant pressure filtration, $P = P_o$ = constant, integrating Eq. 1 yields

$$t = \frac{\eta}{P_o}\left[\overline{\left(\frac{W}{v}\right)(\alpha_{av})_{\Delta p_c}}\frac{V^2}{2} + R_m V\right] \quad (3)$$

and

$$\overline{(w/V)(\alpha_{av})_{\Delta p_c}} = \frac{2}{V^2}\int_o^V \left(\frac{w}{V}\right)(\alpha_{av})_{\Delta p_c} V\,dV \quad (4)$$

From Eq. 2 it is clear that $(\alpha_{av})_{\Delta p_c}$ is a function of Δp_c (and, therefore, t or V). The quantity (w/V), based on mass balance considerations can be expressed as

$$\frac{w}{V} = \rho_s\left[\frac{1}{\phi_o} - \frac{1}{\overline{\phi}}\right]^{-1} \quad (5a)$$

and

$$\overline{\phi} = \frac{1}{L}\int_o^L \phi\,dy \quad (5b)$$

Since the cake is progressively compacted during filtration, $\overline{\phi}$ increases with time and (w/V) is, therefore, a function of time as well. However, both $(\alpha_{av})_{\Delta p_c}$ and (w/V) have their asymptotic values as time increases. Since $\Delta p_c \to p_o$ with increasing time

$$(\alpha_{av})_{\Delta p_c} \to (\alpha_{av})_{p_o} \quad (6a)$$

Similarly, the ultimate value of $\overline{\phi}$ is ϕ at $p_s = P_o$, or

$$\left(\frac{w}{V}\right) \to \rho_s\left[\frac{1}{\phi_o} - \left(\frac{1}{\phi}\right)_{P_o}\right] \quad (6b)$$

Accordingly, if the data were taken under the condition that Eqs. 6a and 6b are valid, one has

$$\overline{\left(\frac{w}{V}\right)(\alpha_{av})_{\Delta p_c}} \cong (\alpha_{av})_{p_o}\frac{\rho_s}{\frac{1}{\phi_o} - \left(\frac{1}{\phi}\right)_{p_o}} \quad (7)$$

With the above approximation, Eq 3 suggests a linear relationship between t/V vs. V, the slope of which can be used to obtain the value of (α_{av}) at $p_s = P_o$. This procedure has been used extensively during the past five decades. Equation 32 of Usher et al. is simply a re-statement of the above.

One can readily apply Eq. 5 to the case where P is increased incrementally. The results are

$$t - t_{i-1} = \frac{\eta}{P_i}\left[(\alpha_{av})_{P_i}\frac{\rho_s}{(1/\phi_o) - (1/\phi)_{P_i}} \times \frac{V^2 - V_{i-1}^2}{2} + R_m(V - V_{i-1})\right]$$

or

$$\frac{t - t_{i-1}}{V - V_{i-1}} = \frac{\eta}{P_i}\left[(\alpha_{av})_{P_i}\frac{\rho_s}{(1/\phi_o) - (1/\phi)_{P_i}} \times \frac{V - V_{i-1}}{2} + R_m\right]$$
$$I = 1, 2, \ldots \quad (8)$$

where P_i is the applied pressure for $t_{i-1} < t < t_i$ and V_{i-1} is the value of V at $t = t_{i-1}$.

Similar to Eq. 3, Eq. 8 suggests a linear relationship between $(t - t_{i-1})/(V - V_{i-1})$ vs V. This relationship, in principle, can be used to obtain the values of (α_{av}) at various P_i's. The derivative of (α_{av}) with P_s then gives the constitutive relationship between α and p_s.

The practicality of applying the suggested alternative, of course, remains to be tested. However, it is obvious that Eq. 8 is based on less assumptions than those given by Usher et al. The linearity of $(t - t_{i-1})/(V - V_{i-1})$ vs. V requires that data used in establishing this relationship be taken at the condition that $\Delta p_c \cong p_i$ and some criteria for assessing data should be developed. The medium resistance R_m can be estimated from the value of dV/dt and $t = 0$. Once R_m is known, the pressure drop across the medium Δp_m can be found as $\Delta p_m = (dV/dt)\eta R_m$ and $\Delta p_c = p_i - \Delta p_m$. The specific cake resistance can then be determined based on data with Δp_c being within a certain precentage of P_i. This procedure can be repeated with increasingly stringent requirement on Δp_c until similar results are obtained. Another requirement for Eq. 8 being valid is the accuracy of the values of t_{i-1} and V_{i-1} to be used. The fact that the pressure increase did not occur instantaneously may, therefore, present problems in applying Eq. 7.

Another issue of our concern is the omission of the wall friction effect in the determination of $P_y(\phi)$. This omission was justified on the basis that the diameter of the test cell used was sufficiently large. Previous studies have shown that the ratio of the cake thickness to the cell diameter L/D and not D alone is the relevant quantity in determining the effect of the wall friction. Shirato et al. (1968) and Tiller et al. (1972) reported that up to 20% of the applied pressure was consumed as wall friction with L/D less than 0.2. More recently, Teoh et al. (2001) found that the wall friction of their multifunction test cell may account for 16–35% of the applied pressure as L/D increased for 0.19 to 0.25. The accuracy of the data reported can be better assessed if similar information can be supplied by the authors.

As a final remark, it was stated in the introductions of both articles that $P_y(\phi)$ and $R(\phi)$ are two fundamental quantities characterizing the cake filtration process. They are, however, directly related to $\phi = \phi(p_s)$ and $\alpha = \alpha(p_s)$ which have been used in cake filtration studies over the past several decades. Because of this correspondence, one cannot argue that one set of the entities is more physically significant than the other. On the other hand, using the constitutive expressions ($\phi = \phi(p_s)$) and $\alpha = \alpha(p_c)$ does have one distinct advantage as they have been used in most filtration studies. The body of information on ϕ and α available in the literature makes it easier to compare the results of future and past investigations.

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Reply:

Tien and Ramarao have raised concerns over the validity of the "stepped pressure filtration" dewaterability characterization technique, validated and utilized in Usher et al. (2001) and de Kretser et al. (2001a). Their concerns relate to the simplistic filtration model applied to justify the use of the gradient dt/dV^2 and the potential for errors due to membrane resistance and wall friction. Tien and Ramarao propose an alternative approach. They also questioned the use of the compressive yield stress $P_y(\phi)$ and the hindered settling function $R(\phi)$ as fundamental dewatering parameters. These concerns are addressed in this letter together with a critical response to the alternative approach.

Stepped pressure filtration

Stepped pressure filtration, utilized in de Kretser et al. (2001a), involves the rapid characterization of suspension compressibility and permability over a range of pressures from one stepped pressure compressibility filtration test and one truncated stepped pressure permeability filtration test. Adoption of the technique enables significant reductions in sample characterization time relative to the traditional method of multiple single pressure filtration tests. In the technique, a compressive yield stress $P_y(\phi)$ data point is determined from the equilibrium solids volume fraction ϕ for each applied pressure in the compressibility filtration test as a measure of compressibility. The gradients dt/dV^2 from each applied pressure in the permeability filtration test are used with $P_y(\phi)$ data to determine hindered settling function $R(\phi)$, data points for each applied pressure, where $R(\phi)$ is inversely related to permeability. The fundamental variation relative to traditional calculation techniques is the use of dt/dV^2 rather than $d(t/v)/dV$. This eliminates the effect of stepped pressures on the gradient, but adds a membrane resistance effect in the early stages of filtration. Justification of the use of dt/dV^2 was presented by Usher et al. (2001).

Simplied filtration modeling. Highly simplified filtration modeling of pressure filtration using a modified Darcy's Law was employed to provide some insight into filtration behavior during stepped pressure filtration. This simplified modeling highlighted the differences between the gradients dt/dV^2 and $d(t/V)/dV$, when a stepped pressure was applied and also when membrane resistance was significant. The modeling simplification of a uniform filter cake solids volume fraction ϕ is not valid for a compressible suspension. However, during constant pressure filtration, fundamental dewatering theory (Landman and White, 1997) predicts that the average filter cake solids volume fraction is constant during a filtration test when membrane resistance is insignificant. As a result, the uniform filter cake simplification is justified for the purposes of understanding filter cake formation behavior.

Influence of membrane resistance and stepped pressure. The calculation method proposed by Usher et al. (2001) does not account for membrane resistance and this has caused concern for Tien and Ramarao. In most experimental characterizations, membrane resistance rapidly diminishes in significance with an increasing extent of filtration V, and can justifiably be ignored. In situations where membrane resistance remains significant, simplified filtration modeling by Usher et al. (2001) suggests that a simple correction can be applied to the gradient dt/dV^2, with the effect of a previous stepped pressure not affecting the gradient dt/dV^2 during filter cake growth. The correction method would involve subtraction of $(\eta\alpha/2PV)$ from the gradient dt/dV^2, given the extent of filtration V, pressure of filtration P, membrane resistance α, and liquor viscosity η.

Proposed alternatives to dt/dV^2. According to the simplified filtration modeling employed in Usher et al. (2001), traditional use of the gradient $d(t/V)/dV$ in single pressure filtration is predicted to be superior to dt/dV^2 because membrane resistance is removed. However, experimental results by Usher et al. (2001) demonstrated that $d(t/V)/dV$ is especially deficient when set pressures are not attained instantaneously and for stepped pressures. For stepped pressures, membrane resistance also has a significant influence on $d(t/V)/dV$. Correction of the gradient $d(t/V)/dV$ to account for membrane resistance, stepped filtration pressures, and the time taken to reach pressure set points would not be experimentally feasible. As such, no practically applicable correction has been proposed.

Tien and Ramarao suggest that, in principle, the gradient $d((t - t_{i-1})/(V - V_{i-1}))/dV$ could be used instead of the gradient dt/dV^2 for permeability determination. A method of this kind was considered in the development of the pressure filtration technique, but was not applied or recommended because of inherent limitations that make it inaccurate and unworkable in a practical sense. The principle limitation is that the time and specific filtrate volume, when the pressure is stepped t_{i-1} and V_{i-1}, are not relevant filtration parameters. This is partly because the first applied pressure and subsequent pressure step changes cannot be applied instantaneously. In addition, the initial stages of filtration after a pressure step involve compaction of the old filter cake before growth of the new filter cake (as was discussed and modeled by Usher et al. (2001)).

Suggestion that the membrane resistance could be determined from the magnitude of the gradient dV/dt at $t = 0$ was also found to experimentally impractical because the set pressure could not be applied instantaneously. Efforts to apply set pressures instantaneously would have increased the likelihood of excessive pressures being applied for a short period and caused the test to produce invalid filtration behavior. As a result of these practical limitations, there were too many cumulative error contributions that could not be adequately accounted for or corrected for when using the gradient $d(t - t_{i-1})/(V - V_{i-1})/dV$. As such, the ideas suggested by Tien and Ramarao have not been developed enough to show potential for practical application. It can be shown that both a modified Darcy's Law (Eq. 27 in Usher et al. (2001)) and conventional cake theory (Eq. 8 in Tien and Ramarao) basically predict the same quadratic t vs. V filtration behavior. As a result, the more complex conventional cake theory has not provided any new insights that the simplistic modified Darcy's Law approach could not predict. As a result, no reasonable justification has been provided for challenging the appropriateness of using dt/dV^2 in stepped pressure filtration dewaterability characterization. Furthermore, it should be pointed out that experimental validation of the stepped pressure approach is presented in both Usher et al. (2001) and de Kretser et al. (2001a).

Wall friction

It is a further criticism of our work that not accounting for wall friction in pressure filtration characterizations will contribute to error in the determination of $P_y(\phi)$. Suggestions that up to 20% of the applied pressure could be consumed as wall friction for cake height (L) to diameter (D) ratios of $L/D < 0.2$ are not discounted outright, but do require further investigation. For the compressibility filtration tests employed in Usher et al. (2001) and de Kretser et al. (2001a), the final L/D ratio was generally in the range 0.17–0.23. An important consideration when evaluating these tests relative to many piston driven filtration tests is that the applied pressure was that measured by a pressure transducer at the piston face rather than an externally applied pressure. For externally applied pressures, the frictional loss between the cylinder wall and the piston has been monitored and shown to consume a very large proportion of the applied pressure, leading to significant errors. In this situation, the wall friction between the cylinder and suspension is usually less significant. The agreement between measurements of the compressive yield stress parameter using a centrifugation technique, piston driven filtration, where the frictional loss of the piston on the wall has been removed, and via sedimentation suggests the concerns raised by Tien and Ramarao are unwarranted. As stated, this assumes that the delivered pressure to the sample is measured, as distinct from the applied pressure. Despite this statement, our fundamental understanding of the effects of wall friction is poor and, until more research delivers this understanding, the influence of wall friction can be minimized by using the largest filter diameter practical for the application.

Fundamental dewatering model

Simplistic modified Darcy's Law theory was used as part of the justification of dt/dV^2. However, the proposed stepped pressure filtration suspension dewaterability characterization technique does not utilize this simplified modeling, but uses the fundamentally based calculation method provided by Landman et al. (1999). de Kretser et al. (2001a) considers this process in detail. Additional verification of the stepped pressure approach is also presented by de Kretser et al. (2001b) using the full Landman and White (1997) theory adapted for stepped pressure tests. In this article both a permeability and compressibility stepped pressure test are compared and agree with full model predictions with no assumption of constant filter cake solids concentration.

The suggestion that conventional cake theory could be used for data treatment ignores the developing trend of using $P_y(\phi)$ and $R(\phi)$ as standard measures of dewaterability. These parameters are used because of their applicability to the prediction and characterization of a wide range of dewatering operations, not just filtration. These operations include batch settling, continuous settling, and centrifugation. This enables application of the same dewaterability data to the prediction of industrial filtration, thickening, centrifugation and tailings dam consolidation operations.

Other issues

An important issue relating to stepped pressure filtration involves determining when filtration has ceased for an applied pressure in a compressibility filtration test. This is addressed by de Kretser et al. (2001b) based on the description of cake compression provided in Landman and White (1997). Theoretical and experimental application of this theory has demonstrated the practicality of predicting the final solids concentration of an incomplete pressure filtration test. Application to stepped pressure filtration compressibility tests has the potential to further reduce characterization time and also to improve the accuracy of $P_y(\phi)$ data.

Conclusions

To sum up, the first principle predictive capability for filtration processes (invoking the theory of Landman et al. (1999)), using the parameters measured with the stepped pressure filtration technique, demonstrate clearly that the method is neither flawed nor ineffective. The technique was developed to overcome many of the problems inherent in dewaterability characterization tests, namely that the characterization process was both slow and nonautomated. The current method, when applied to filtration characterization apparatus, allows portable, automated, in field, nonempirical dewaterability characterization in a few hours (depending on the pressure regime of interest). As stated, the parameters are applicable to a wide range of dewatering operations, not just filtration. The use of the simplified Darcian model to analyze the dt/dV^2, as distinct from $d(t/V)/dV$ approach taken here, shows clearly that although membrane resistance represents a small error in this case, it can be corrected easily and retrospectively. Also, problems associated with noninstantaneous pressure stepping and the time taken for cake rearrangement after a pressure step is eliminated. The method of Tien and Ramarao does not achieve this goal.

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